Matematika 1

Monday, December 16, 2024 6:42 PM

1. $\int \frac{e^x + e^{mx} - e^{5x} - n}{e^{2x}}\, dx$

\# $m = 3 \quad n = 93$

$\int \frac{e^x + e^{3x} - e^{5x} - 93}{e^{2x}}\, dx$

$\int \frac{e^x}{e^{2x}}\, dx + \int \frac{e^{3x}}{e^{2x}}\, dx - \int \frac{e^{5x}}{e^{2x}}\, dx - \int \frac{93}{e^{2x}}\, dx$

$\int e^{-x}\, dx + \int e^x\, dx - \int e^{3x}\, dx - 93 \int e^{-2x}\, dx$

$-\frac{1}{e^x} + e^x - \frac{e^{3x}}{3} + \frac{93}{2e^{2x}} + C$

$\int e^{-2x}\, dx$

$u = -2x$ $\quad -\frac{93}{2}\int e^u\, du$

$du = -2\, dx$

$-\frac{du}{2} = dx$ $\quad -\frac{93}{2e^{2x}}$

2. $\int \frac{x^2}{\sqrt{9 - x^2}}$ $\quad x = 3 \sin t$ $\quad dx = 3 \cos t\, dt$

$\sqrt{x^2 - a^2} = a \sec t$

$\sqrt{a^2 + x^2} = a \tan t$

$\int \frac{9 \sin^2 t}{\sqrt{9 - 9\sin^2 t}}$

$\int \frac{9 \sin^2 t}{\sqrt{9(1 - \sin^2 t)}}$

$\cos^2 t + \sin^2 t = 1$

$\cos^2 t = 1 - \sin^2 t$

$\int \frac{9 \sin^2 t}{\sqrt{9 \cos^2 t}}$

$\int \frac{9 \sin^2 t}{3 \cos t} \cdot 3 \cos t \cdot dt$

$9 \int \sin^2 t\, dt$

$\cos 2t = 1 - 2\sin^2 t$

$\cos 2t - 1 = -2 \sin^2 t$

$\frac{1 - \cos 2t}{2} = \sin^2 t$

$\frac{9}{2} \int 1 - \cos 2t\, dt$

$\frac{9}{2} t - \frac{9}{2} \frac{\sin 2t}{2} + C$

$\frac{9}{2} t - \frac{9}{2} \sin t \cos t + C$

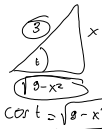
$\frac{9}{2}(t) - 9 \sin t \cos t + C$

$\frac{x}{3} = \sin(t)$

$\sin^{-1}\left(\frac{x}{3}\right) = t$

$x = 3 \sin t$

$\frac{x}{3} = \sin t \longleftrightarrow \arcsin\left(\frac{x}{3}\right) = t$



$\cos t = \frac{\sqrt{9 - x^2}}{3}$

$\frac{9}{2} \arcsin\left(\frac{x}{3}\right) - \frac{3}{2} \cdot \frac{x}{3} \cdot \frac{\sqrt{9-x^2}}{3}$

$= \frac{9}{2} \arcsin\left(\frac{x}{3}\right) - x\sqrt{9 - x^2} + C$

atau

$= \frac{9 \arcsin\left(\frac{x}{3}\right) - 2x\sqrt{9 - x^2}}{2} + C$

$\sin^{-1} t = \arcsin(t)$

3. $\int \frac{4x - m}{x^3 - 8x^2 + 20x - 16}\, dx$

\# $m = 8$

b. $\int \cos x\, dx$

b. $\int_0^3 \frac{1}{\sqrt{x+1}}\, dx$

$u = \sqrt{x+1}$

$u^2 - 1 = x$

$2u\, du = dx$

$\int_1^2 \frac{1}{u} \cdot 2u \cdot du$

$\int_0^3 2\, du$

$\left(2\sqrt{x+1}\right)_0^3$

$4 - 2 = 2$

1. $\int \frac{x}{x^2 - 1}\, dx$

$\frac{x}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$

$x = Ax + A + Bx - B$

$x = (A+B)x + (A-B)$

$A - B = 0$ $\quad 1 = 2A$

$A = B$ $\quad A = \frac{1}{2}$

$B = \frac{1}{2}$

$\frac{\ln|x-1|}{2} + \frac{\ln|x+1|}{2} + C$

$u = x^2 - 1$

$du = 2x\, dx$

$\frac{du}{2x} = dx$

$\int \frac{x}{u} \frac{du}{2x}$

$\frac{\ln|x^2 - 1|}{2} + C$

2. a. $\int x \ln(x)\, dx$

$u = \ln(x)$ $\quad dv = x\, dx$

$du = \frac{1}{x}\, dx$ $\quad v = \frac{x^2}{2}$

$\frac{x^2 \ln(x)}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x}\, dx$

$\frac{x^2 \ln(x)}{2} - \frac{1}{2} \int x\, dx$

$\frac{x^2 \ln(x)}{2} - \frac{x^2}{2} + C$

$$\int \frac{4x-8}{x^3-8x^2+20x-16}\,dx$$

$x^3-8x^2+20x-16=0$

$x=2$

$8-32+40-16$

$-24-16+40$

$=-40+40$

$=0$

	1	−8	20	−16
2		2	−12	16
	1	−6	8	0

$$\int \frac{4x-8}{(x-2)(x^2-6x+8)}$$

$$\int \frac{4(x-2)}{(x-2)(x-4)(x-2)}\,dx$$

$$\int \frac{1}{(x-4)(x-2)}\,dx$$

$$\frac{1}{(x-4)(x-2)} = \frac{A}{x-4} + \frac{B}{x-2}$$

$1 = A(x-2) + B(x-4)$

$1 = Ax - 2A + Bx - 4B$

$1 = (A+B)x + (-2A-4B)$

$A+B=0$ $\quad -2A-4B=1$

$A=-B$ $\quad -2(A-2B)=1$

$A-2B=-2$

$-B-2B=-2$

$-3B=-2$

$B=\frac{2}{3}$

$A=-B$

$=-\frac{2}{3}$

$$-\frac{2}{3}\int \frac{1}{x-4}\,dx + \frac{2}{3}\int \frac{1}{x-2}\,dx$$

$$-\frac{2}{3}\ln|x-4| + \frac{2}{3}\ln|x-2| + c$$

$u = \sin^2 x \qquad dv = \cos x\,dx$

$du = 2\sin x\cos x \qquad v = -\sin x\,dx$

$du = \sin 2x\,dx$

$uv - \int v\,du$

$-\sin^3 x + \int \sin x \cdot \sin 2x$

$$+\int \sin x \cdot 2\cdot \sin x\cdot \cos x\,dx$$

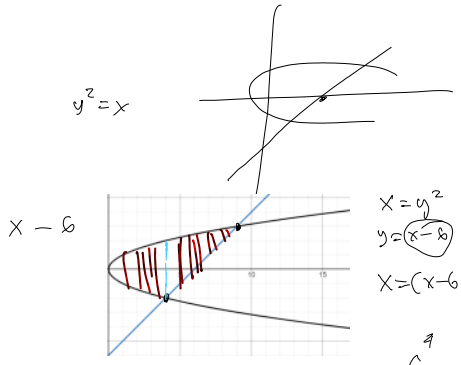
$-\sin^3 x + 2\int \sin^2 x\cdot\cos x\,dx = \int \sin^2 x\cos x\,dx$

$-\sin^3 x = -\int \sin^2 x\cos x\,dx$

$\frac{\sin^3 x}{3} + C$

4

1. Tentukan integral dari
   a. $\int \frac{x}{x^2-1}\,dx$ b. $\int_0^3 \frac{1}{\sqrt{x+1}}\,dx$.
   Soal 1 (20% ; Pembelajaran 9)
2. Carilah integral berikut dengan menggunakan teknik integrasi yang sesuai.
   a. $\int x(\ln x)\,dx$ c. $\int \frac{x}{\sqrt{x^2+1}}\,dx$
   b. $\int \sin^2 x\cos x\,dx$ d. $\int \frac{x+1}{x^2+x-2}\,dx$
   Soal 2 (40% ; Pembelajaran 10, 11, 12)
3. Daerah D dibatasi oleh kurva $y^2=x$, garis $y=x-6$, dan sumbu y. Sketsalah daerah D dan carilah luasnya serta tentukan volumenya jika daerah D diputar terhadap garis $x=0$ satu kali putaran!
   Soal 3 (40% ; Pembelajaran 13, 14, 15)



$x=y^2$

$y=x-6$

$x=(x-6)^2$

$x=(x-6)^2$

$x=x^2-12x+36$

$0=x^2-13x+36$

$(x-9)(x-4)$

$$\int_0^4 y^2\,dy + \int_4^9 x^2-12x+36$$

$$\left(\frac{y^3}{3}\right)_0^4 + \left(\frac{x^3}{3}-6x^2+36x\right)_4^9$$

$$64 + \left(243-486+324\right) - \left(\frac{64}{3}-96+144\right)$$

$\frac{64}{3}+\frac{144}{3}=\frac{144}{200}$

$64+81-\frac{208}{3}$

4. $y = x^3-3x^2-x+3$

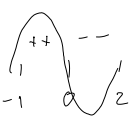
$y = x^3-3x^2-x+3$

$y' = 3x^2-6x-1$ $\qquad y'' = 6x-6$

$0 = 3x^2-6x-1$ $\qquad 0 = 6x-6$

$6x=6$

$x=1$



$x=1$ $\qquad x=-1$

$0 = 3\cdot 1+6-1$

$x = 1$

$0 \geq 3 - 6 - 1$

$0 \geq -4$

$1 = -1$

$0 = 3 \cdot 1 + 6 - 1$

$0 = 3 + 6 - 1 = 8$

$x = 1$

$x^3 - 3x^2 - x + 3$

$1 - 3 - 1 + 3$

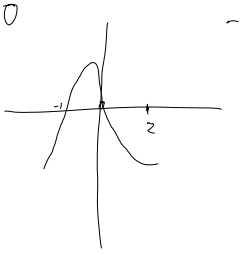
$= 0$

$x = 2$

$8 - 12 - 2 + 3$

$-4 - 2 + 3$

-3 //



ATAS

$-1 \rightarrow 1$

BAWAH

$1 \rightarrow 2$

$$\int_{-1}^{1} (x^3 - 3x^2 - x + 3) - \int_{1}^{2} (x^3 - 3x^2 - x + 3)$$

$$\left(\frac{x^4}{4} - x^3 - \frac{x^2}{2} + 3x \right)_{-1}^{1}$$

$$\left(\left(\frac{1}{4} - 1 - \frac{1}{2} + 3 \right) \right) - \left(\frac{1}{4} + 1 - \frac{1}{2} - 3 \right)$$

$$\left(\frac{1 - 4 - 2 + 12}{4} \right) - \left(\frac{1 + 4 - 2 - 12}{4} \right)$$

$$\frac{7}{4} - \left(\left(-\frac{9}{4} \right) \right)$$

$$= \frac{16}{4}$$

$$\left(\frac{x^4}{4} - x^3 - \frac{x^2}{2} + 3x \right)_{1}^{2}$$

$$\left(\frac{16}{4} - 8 - \frac{4}{2} + 6 \right) - \left(\frac{1}{4} \right.$$

$$\left(4 - 8 - 2 + 6 \right) - \frac{7}{4}$$

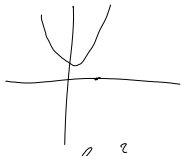
$$-\frac{7}{4}$$

$$\frac{16}{4} - \left(-\frac{7}{4} \right)$$

$\frac{23}{4}$ satuan luas

//

1. $y = x^2$



$$\int_1^2 x^2 \, dx$$

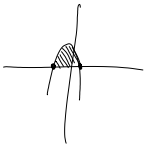
$$\left[\frac{x^3}{3} \right]_1^2 = \left(\frac{8}{3} \right) - \left(\frac{1}{3} \right)$$

$$= \frac{7}{3} \text{ satuan luas}$$

Hitung luas daerah yang dibatasi oleh :

1. Kurva $y = x^2$, sumbu-x, garis $x = 1$ dan garis $x = 2$
2. Kurva $y = x^2 + 2$, sumbu-x, garis $x = -1$ dan garis $x = 1$
3. Kurva $y = -x^2 + 3$, sumbu-x, garis $x = -2$ dan garis $x = 1$
4. Kurva $y = -x^2 - x + 2$ dan sumbu-x
5. Kurva $y = x^2 - 3x + 2$ dan sumbu-x
6. Kurva $y = x^2 - 1$, ruas sumbu-x dari $x = -1$ sampai $x = 2$ dan garis $x = 2$

4. $y = -x^2 - x + 2$



$0 = -x^2 - x + 2$

$0 = x^2 + x - 2$

$(x+2)(x-1)$

$x = -2 \vee x = 1$

$$\int_{-2}^{1} -x^2 - x + 2 \, dx$$

$$\left(-\frac{x^3}{3} - \frac{x^2}{2} + 2x \right)_{-2}^{1}$$

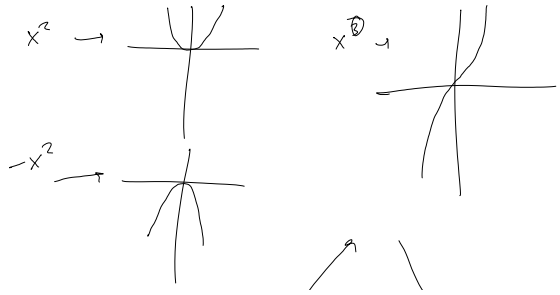
$$\left(-\frac{1}{3} - \frac{1}{2} + 2 \right) - \left(\frac{8}{3} - \frac{4}{2} - 4 \right)$$

$$\left(\frac{-2 - 3 + 12}{6} \right) - \left(\frac{16 - 12 - 24}{6} \right)$$

$$\left(\frac{-7}{6} \right) - \left(-\frac{20}{6} \right)$$

$$-\frac{7}{6} + \frac{20}{6} = \frac{13}{6} \text{ satuan luas}$$

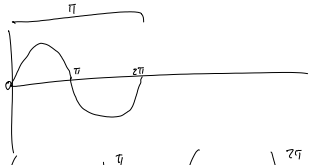
$-\frac{7}{6} + \frac{20}{6} = \frac{13}{6}$ satuan luas



9. Kurva $y = \sin x$, ruas sumbu-x dari $x = 0$ sampai $x = 2\pi$

10. Kurva $y = \cos x$, ruas sumbu-x dari $x = 0$ sampai $x = \frac{3}{2}\pi$ dan garis $x = \frac{3}{2}\pi$.

$y = \sin x$



$\left(-\cos x\right)\Big|_0^{\pi} - \left(-\cos x\right)\Big|_{\pi}^{2\pi}$

$1 + 1 = 2 - (-2)$

$-(\cos \pi)$ $= 1$

-1 $(-\cos 2\pi) - (-\cos \pi)$

$-1 - (1)$